MS&E 319: Matching Theory
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Some open problems in the area of online matching

Problem 1. Here is a simple model for matching in a dynamic environment. Imagine we have n types of agents arriving and departing the system according to a simple poisson process. Let w_i^t be the number of agents of type i present in the system at time t.

Denote the set of all agents present at time t by N^t. Let $v_{(i,j)}$ be the utility obtained by matching an agent of type i to an agent of type j. Furthermore, let $x_{(i,j)}^t$ denote the number of matched agents of type i to type j at time t. The optimum online policy can be obtained by solving the following program for x:

$$
\begin{aligned}
OPT_t(\mathbf{w^t}) = \max_x \sum_{i,j \in N^t, i<j} v_{(i,j)} x_{(i,j)}^t + \sum_{\mathbf{w}^{t+1}} P(\mathbf{w}^{t+1} | \mathbf{r}^t) \mathrm{OPT}_{t+1}(\mathbf{w}^{t+1}) \\
\text{subject to.} \sum_{j \in [n], j \neq i} x_{(i,j)}^t + r_i^t = w_i^t \qquad \forall i \in [n]
\end{aligned}
$$

where $\mathbf{r}^t$ is a vector representing the number of unmatched agents of each type at the end of time t. The second sum is over all possible values of $\mathbf{w}^{t+1}$.

In general, finding the optimum solution of the above program is PSPACE-complete. What is the best possible approximation ratio? Are there ways to approximate the value function using techniques in reinforcement learning?

Problem 2. Consider the model presented in Ashlagi et al (EC 2019). In their model they prove that when the arrival order of agents is chosen uniformly at random, a batching algorithm which computes a maximum weight matching every $(d+1)$ periods, is 0.279 competitive. It is conjectured that the correct competitive ratio of this algorithm is 0.5 and is tight. Can you prove or disprove this conjecture?

Problem 3. Would it be possible to come up with a decomposition similar to Gallai-Edmonds for online matching with random arrival and departure of vertices?

Problem 4. Develop a model for matching with learning, where the value of a matching two agents to each other is a function of the features of the agents. The platform does not have direct access to the features but it can learn about them by proposing and observing the value of tentative matchings.

Problem 5. Consider the model presented in Akbarpour et al. (2017). In their paper, the arrival and departure of agents are modeled with a poisson process. Would it be possible to improve their results if we have more information about the departure time of each agent?